LINEAR ARRAY of IDENTICAL ELEMENTS

Controls that can be used to shape the overall pattern of the antenna:

1. The geometrical configuration of the overall array (linear, circular, rectangular, spherical, etc.)

2. The relative displacement between the elements

3. The excitation amplitude of the individual elements

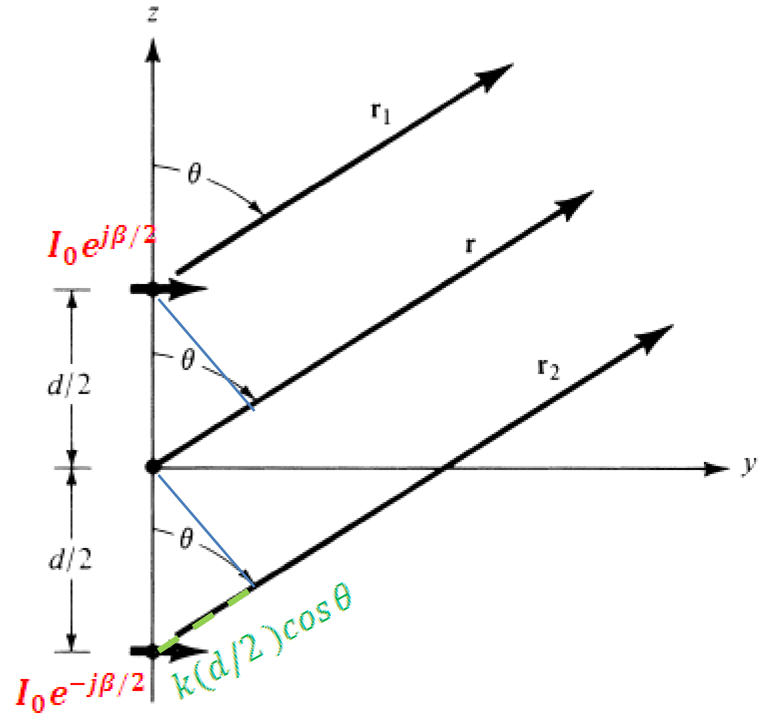
4. The excitation phase of the individual elements

5. The relative pattern of the individual elements

TWO-ELEMENT ARRAY

For two infinitesimal horizontal dipoles positioned along the *z*-axis

$$\mathbf{E}_t = \mathbf{E}_1 + \mathbf{E}_2 = \hat{\mathbf{a}}_\theta j\eta \frac{kI_0 l}{4\pi}\left\{\frac{e^{-j[kr_1-(\beta/2)]}}{r_1}\cos\theta_1 + \frac{e^{-j[kr_2+(\beta/2)]}}{r_2}\cos\theta_2\right\}$$



(b) Far-field observations

$$E_t = \hat{a}_\theta j\eta \frac{kI_0 l e^{-jkr}}{4\pi r}\cos\theta\left[e^{+j(kd\cos\theta+\beta)/2} + e^{-j(kd\cos\theta+\beta)/2}\right]$$

$$E_t = \hat{a}_\theta j\eta \frac{kI_0 l e^{-jkr}}{4\pi r}\cos\theta\left\{2\cos\left[\frac{1}{2}(kd\cos\theta+\beta)\right]\right\}$$

$$\text{E(total)} = [\text{E(single element at reference point)}] \times [\text{array factor}]$$

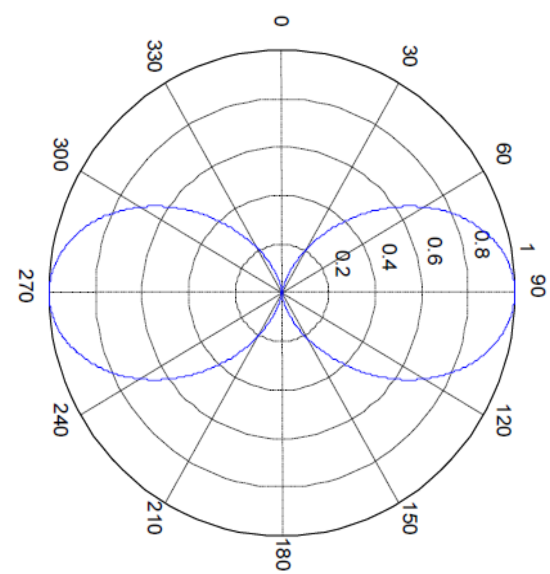
Arrays of two isotropic point sources (ARRAY FACTOR PATTERN)

Case 1 same amplitude and phase (*β=0*) and for d=λ/2 Kd/2=π/2

(AF)n= $\cos[(kd/2)\cos\theta] = \cos[(\pi/2)\cos\theta]$

Max at $\frac{\pi}{2}\cos\theta_m = m\pi$, $m = 0,1,2,\ldots$ $\theta_m = \cos^{-1}(0) = \frac{\pi}{2}, -\frac{\pi}{2}$

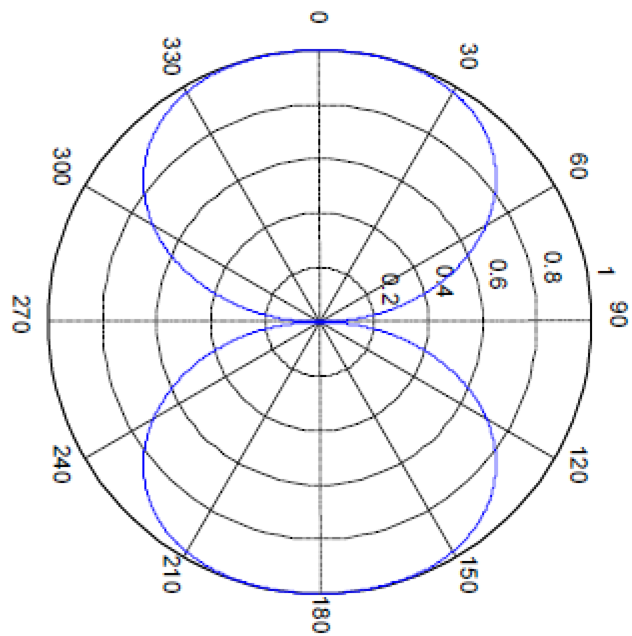
Nulls at $\frac{\pi}{2}\cos\theta_n = \pm(2m+1)\pi/2$ $\theta_n = \cos^{-1}(\pm 1)$**=0,π**



- **Case 2 same amplitude and opposite phase (β=180) and for d=λ/2 Kd/2=π/2**

$$\left|(AF)_n\right| = \cos\left(\frac{kd}{2}\cos\theta + \pi/2\right) = \sin\left(\frac{kd}{2}\cos\theta\right)$$

θ	0	10	20	30	40	50	60	70	80	90
AF_n	1	.999	.995	.978	.933	.847	.707	.5	.269	0

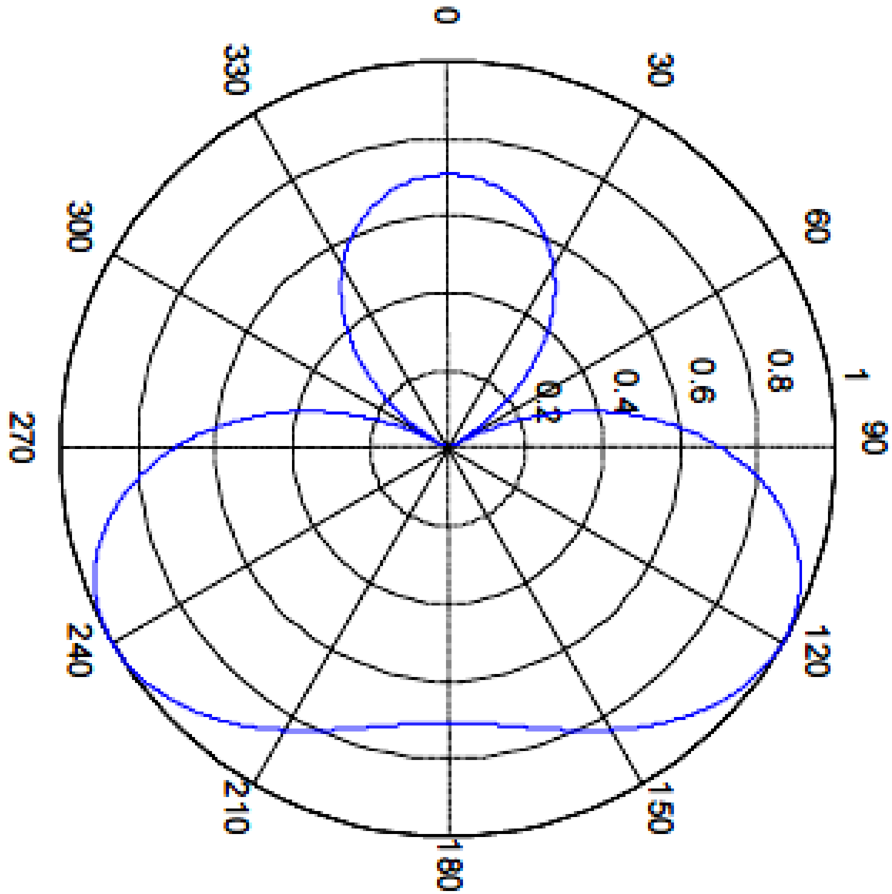


***Changing phase of source currents shift to 180 change AF pattern as seen in Fig*.**

- **Case 3 (a) same amplitude and quadrature phase (β/2=π/4) and for d=λ/2 Kd/2=π/2**

$$\left|(AF)_n\right| = \cos\left(\frac{\pi}{2}\cos\theta + \pi/4\right)$$

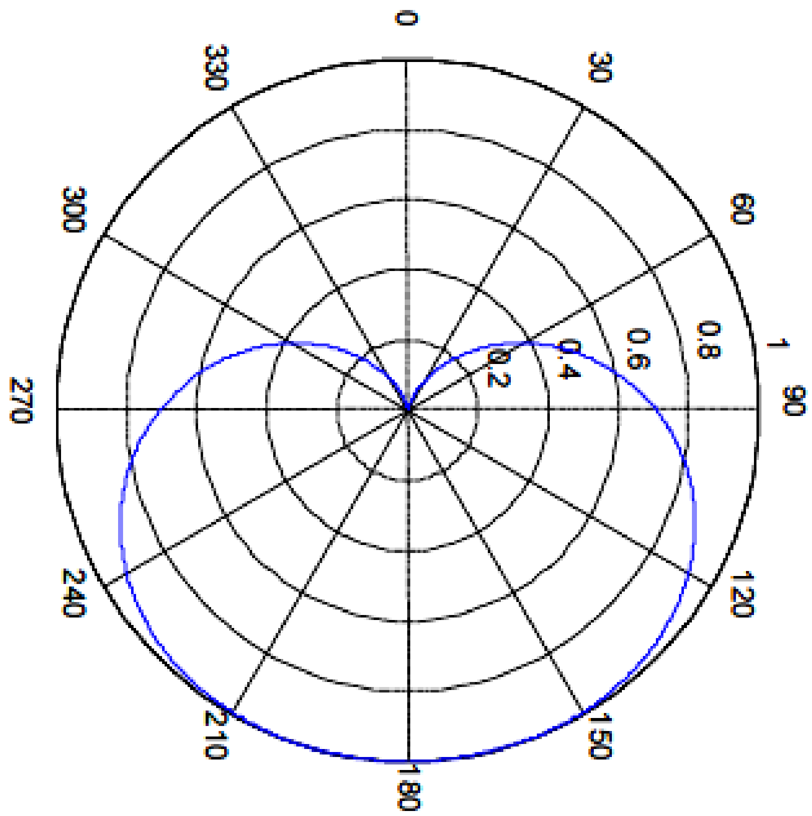
θ	0	10	20	30	40	50	60	70	80
$\|AF_n\|$	.707	.69	.637	.543	.406	.22	0	.245	.49
θ	90	100	110	120	130	140	150	160	170
$\|AF_n\|$	.707	.87	.969	1	.975	.913	.839	.77	.723
θ	180								
$\|AF_n\|$	.707								



Most radiation directed toward lower half

- **Case 3 (b) same amplitude and quadrature phase (β/2=π/4) and for d=λ/4 Kd/2=π/4**

$$\left|(AF)_n\right| = \cos\left(\frac{\pi}{4}\cos\theta + \pi/4\right)$$



Can we explain max occurred at 180 and nulls at 0 in terms of phase accumulation of phases Generated by array elements at observation point, …this illustrated at next slide.

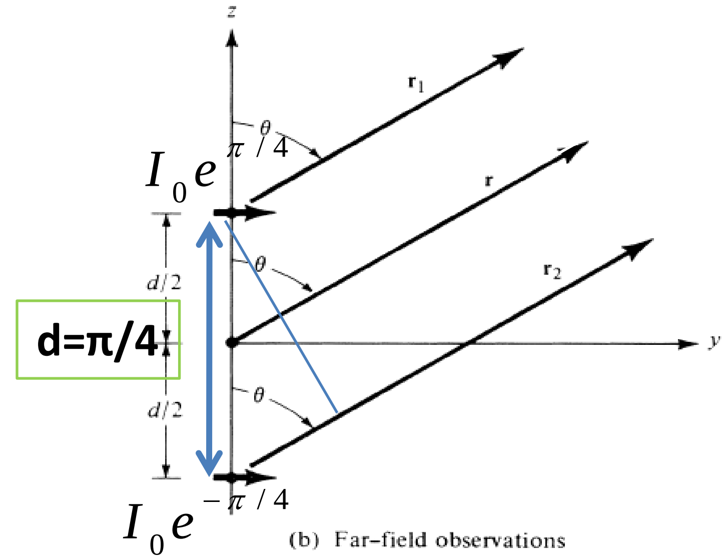
Phase accumulation:

We will notice at 0^0 the two fields at the point #1

Are out of phase thus destructive , minima exist

And at 180 the two fields are in phase at point #1

Constructive thus maxima exist



(b) Far-field observations

$$Etot = E1 + E2$$

take referece point at #1 element to view E2 phases as arrived at point #1

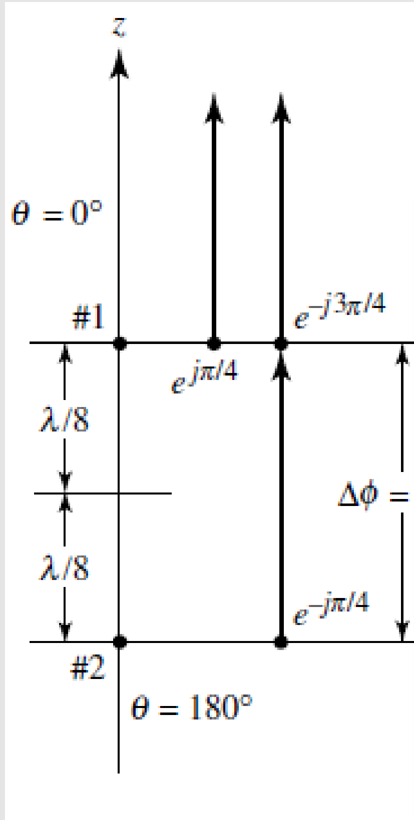
$$\mathrm{Etot} = \mathrm{E_o e^{-jkr1}}\, e^{j\pi/4} + \mathrm{E_o e^{-jkr2}}\, e^{-j\pi/4} = \mathrm{E_o e^{-jkr1}}\left(e^{j\pi/4} + e^{-jkd\cos\theta}\, e^{-j\pi/4}\right)$$

$$d = \lambda / 4, \mathrm{kd} = \pi/2$$

$$\mathrm{Etot} = \mathrm{E_o e^{-jkr1}}\left(e^{j\pi/4} + e^{-j(\pi/2)\cos\theta} . e^{-j\pi/4}\right)$$

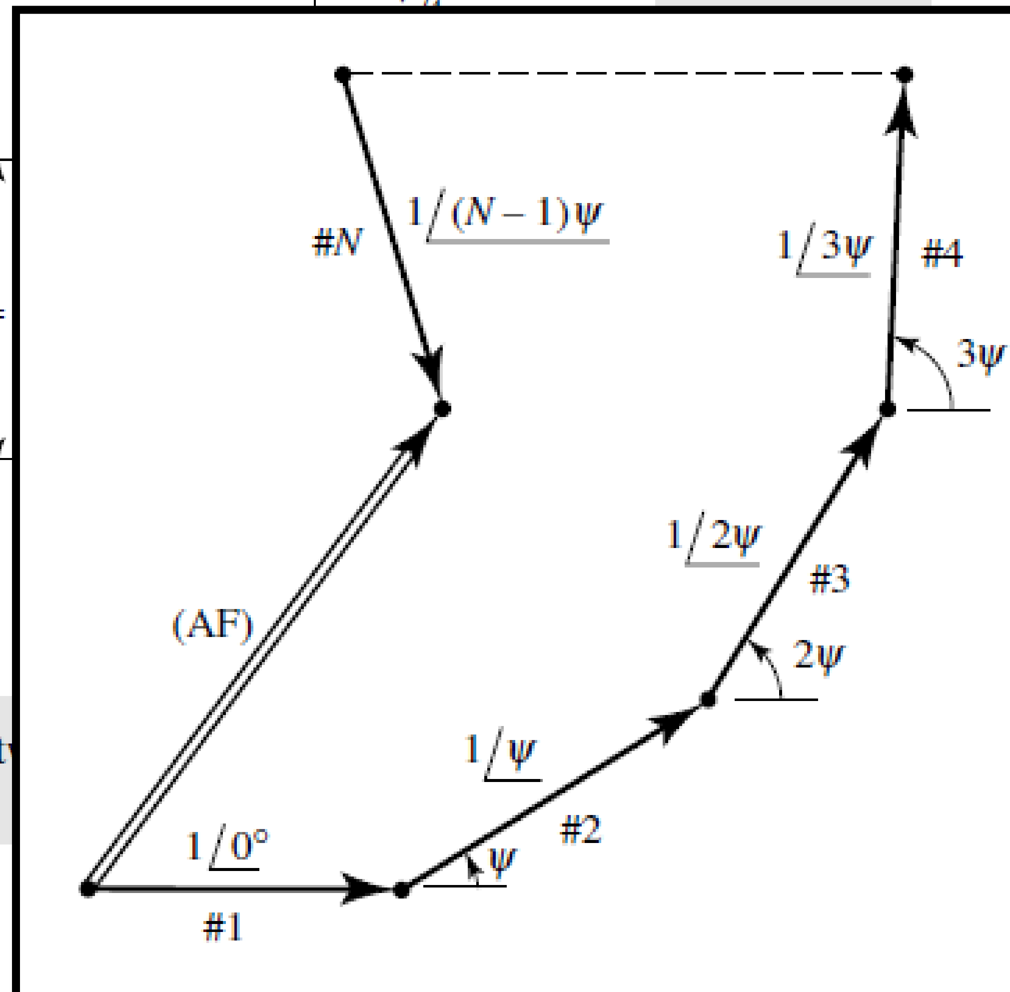
$$at\ \theta = 0 \rightarrow \mathrm{Etot} = \mathrm{E_o e^{-jkr1}}\left(e^{j\pi/4} + e^{-j(\pi/2)} . e^{-j\pi/4}\right) = \mathrm{E} = \mathrm{E_o e^{-jkr1}}\left(e^{j\pi/4} + e^{-j3\pi/4}\right)$$

$$at\ \theta = 180 \rightarrow \mathrm{Etot} = \mathrm{E_o e^{-jkr1}}\left(e^{j\pi/4} + e^{j(\pi/2)} . e^{-j\pi/4}\right) = \mathrm{E_o e^{-jkr1}}\left(e^{j\pi/4} + e^{j\pi/4}\right)$$



(a) $\theta = 0°$ direction

Figure 6.2 Phase accumulation for t
and 180°.

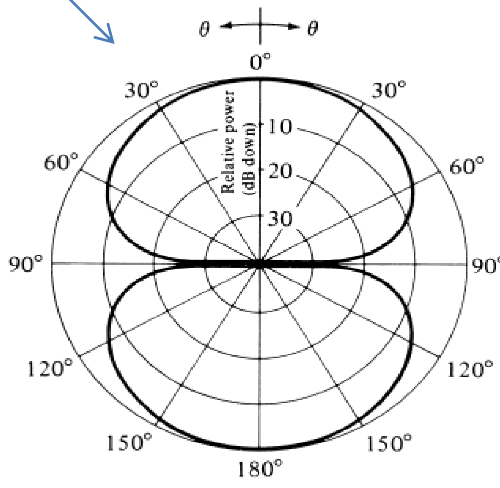


(b) Phasor diagram

Pattern multiplication rule Total= element x AF= element(dB)+ AF(dB)

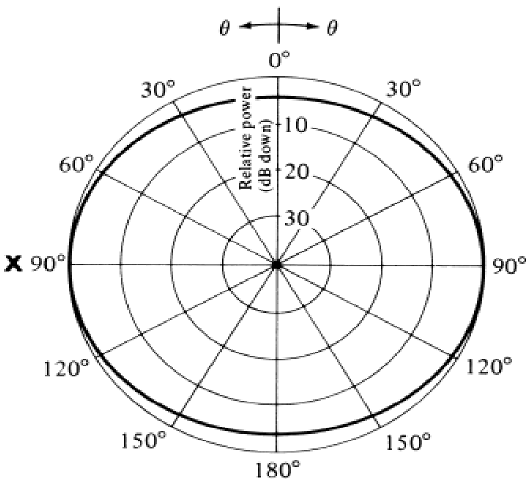
For two identical elements ,same amplitude ,phase ($\beta=0$) , and d=λ/4 ;   Kd/2=π/4

$$\left|(E_{ref})_n\right| = \cos\theta \qquad \text{X} \qquad \left|(AF)_n\right| = \cos\left(\frac{\pi}{4}\cos\theta\right)$$



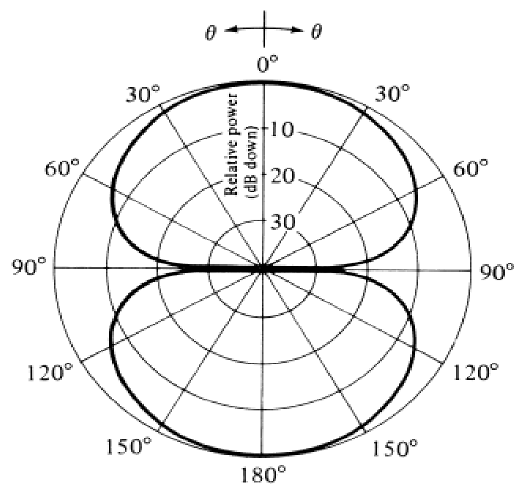
Element

X



Array factor

=



Total